## Relaxation Time & Mean free path

Let us suppose that as soon as the velocity of the directional motion of the electrons attains a constant value $v_d$, the field is turned off. This velocity starts diminishing as a result of collisions of thee electrons with the phonons, impurities and lattice imperfectio and the electron gas ultimately return to an equilibrium state. Such a process leading to the establishment of equilibrium in a system is termed as relaxation process. Thus for $E=0$, the eqn. (3) becomes

$$\frac{d v_d(t)}{dt} = -\frac{v_d(t)}{\tau}$$

so that

$$v_d(t) = v_d \exp\left[-\frac{t}{\tau}\right] \qquad (5)$$

where $v_d(t)$ is the velocity of the directional motion of the electrons and $t$ is the time after the field is turned off. In eqn. (5), $\tau$ characterized the rate at which the equilibrium state of a system is reached; smaller is the $t$ sooner the system reaches to equilibrium state. For $t=\tau$, the velocity of the directional motion decreases by $1/e$ of its initial value. For pure metals, $\tau \simeq 10^{-14}$ s.

The motion of an electron in a crystal may be conveniently described in terms of mean free path. By analogy with the kinetic theory of gases one may presume that an electron in a crystal moves along a straight line until it collides with the lattice imperfection collisions is taken as the mean free path of the electron. At room temperature, since the velocity imparted to the electrons by an applied electric field is much smaller thermal velocity, the time $\tau$ taken by the electrons is travelling the distance $\lambda$ will thus be decided not by the drift velocity due to the field but by the average velocity $v$, due to random thermal motion. Therefore,

$$\tau = \frac{n\lambda}{v} \quad \text{—— (6)}$$

where $n$ is the number of collisions that are required to nullify the directional velocity completely.

## Electrical conductivity and Ohm's Law

Ohm's law is the most established experimental law relating to the conduction in metals and can be used to test the validity of the ~~electrons~~ theory of electrical conductivity.

Knowing the drift velocity of the electrons, it is easy to calculate the current density and hence the conductivity of a metal. For the purpose, let us consider a cylindrical conductor of length $v_d$ and area of cross-section of unity as shown in fig. (8). suppose it contains

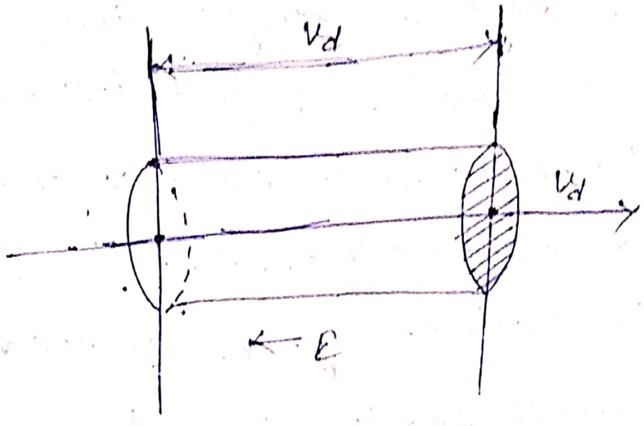


fig. (8) calculation of current density.

N electrons per unit volume. Imagine any section of the conductor and count the number of charges passing through this section per second. obviously, it will be equal to all the electrons inside this cylinder of volume $1 \cdot v_d$. Therefore, a current flowing through the conductor with a density

$$I = N(1 \cdot v_d)e = Ne\left(\frac{eE\tau}{m}\right) \quad \text{from eq}^n \text{ (4)}$$

$$= \left(\frac{Ne^2\tau}{m}\right)E \quad \text{—— (7)}$$

This is at once recognizable as Ohm's law ($I = \sigma E$) where the conductivity $\sigma$ is given by

$$\sigma = \left(\frac{Ne^2\tau}{m}\right) = Ne\mu \quad \text{—— (8)}$$

where $\mu = e\tau/m$ is called the carrier mobility and is defined as the average drift velocity per unit electric field, i.e.

(7)

$$\mu = \frac{v_d}{E} = \frac{e\tau}{m}$$

and the resistivity $\rho$ is given by

$$\rho = \frac{m}{Ne^2\tau} \quad \text{—— (9)}$$

Eqn. (7) can be easily understood as follows: we expect the charge transported in the medium to be proportional to the charge density ($n = Ne$), the factor ($e/m$) enters because the acceleration in a given electric field is proportional to e and inversely proportional to m (eqn. (1)) and the time $\tau$ describes the time during which the field acts on the carrier. This equation is of fundamental importance. The electrical conductivity $\sigma$ depends on two factors, the number n of carriers per unit volume and their mobility $\mu$. The dependence of these quantities particularly on temp. provides the basic understanding of the electrical properties of materials. For example in metals, n is constant and $\mu$ varies relatively slowly with temp. In semiconductors, the exponential dependence of n is of primary importance while in some insulators, it is the exponential dependence of $\mu$ on temp. that is significant while n is constant. An understanding of the relative contributions of n & $\mu$ to $\sigma$ enables us to explain the whole spectrum of values of $\sigma$